Engineering Notes

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Buckling of Orthotropic, Curved, Sandwich Panels in Shear and Axial Compression

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Nomenclature

a,b = axial length and circumferential width of panel
a_{mn},b_{mn},c_{mn} = Fourier coefficients of w, Q_x and Q_y
c = core depth
D_x,D_y,D_{xy} = flexural and twisting rigidities of panel
E_x,E_y = axial and circumferential Young's moduli of facings
G_{xy} = in-plane shear modulus of orthotropic facings
G_{xz},G_{yz} = thickness shear moduli in the xz and yz planes
H_1,H_2,H_3 = left-hand sides of equilibrium equations, Eqs. (1-3) in Ref. 2
K_s = dimensionless shear buckling coefficient
M_x,M_y = bending stress couples acting on edges cut by x = const., y = const.
m,n = axial and circumferential wave numbers
$N_x,(N_x)_{cr}$ = axial compressive stress resultant and its critical value
$N_{xy},(N_{xy})_{cr}$ = in-surface shear stress resultant and its critical value
Q_x,Q_y = thickness-shear stress resultants in xz and yz planes
R = radius of panel at middle surface
t = facing thickness of sandwich panel; thickness of thin panel
w = normal deflection
x,y = orthogonal curvilinear coordinates on panel middle surface in the axial and circumferential directions
γ_x,γ_y = thickness-shear strains in xz and yz planes
ν_{xy},ν_{yx} = Poisson's ratios corresponding to loading in x and y directions
ν = Poisson's ratio of thin, homogeneous, isotropic panel

BUCKLING is usually the primary structural-design criterion for curved or flat, sandwich or thin, orthotropic (composite-material) or isotropic, rectangular panels, which are widely used in aircraft structures. Buckling under axial compression has been treated extensively, c.f. Ref. 1, but buckling under edge shear has received much less attention.[2] Reference 2 considered simply-supported edges only. The present work is an extension of Ref. 2 to include: 1) clamped-edge panels under pure edge shear, and 2) both simply-supported and clamped panels subjected to combined shear and axial compression.

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Linear Buckling Analysis

The two sets of boundary conditions considered here are:

a) All edges simply-supported, with thickness-shear strain prevented by edge beams:

$$w = M_x = \gamma_y = 0 \text{ along } x = 0,a \tag{1}$$

$$w = M_y = \gamma_x = 0 \text{ along } y = 0,b \tag{2}$$

b) All edges clamped, with thickness-shear strain prevented by edge beams:

$$w = w,_x - \gamma_x = \gamma_y = 0 \text{ along } x = 0,a \tag{3}$$

$$w = w,_y - \gamma_y = \gamma_x = 0 \text{ along } y = 0,b \tag{4}$$

Boundary conditions (1) and (2) are satisfied by the single double trigonometric series used in Ref. 2. Boundary conditions (3) and (4) are satisfied by the following necessarily more complicated series:

$$w = \sum\sum a_{mn}[\cos(m-1)(\pi x/a) - \cos(m+1)(\pi x/a)] \cdot[\cos(n-1)(\pi y/b) - \cos(n+1)(\pi y/b)] \tag{5}$$

$$Q_x = \sum\sum b_{mn}[\sin(m-1)(\pi x/a) - \sin(m+1)(\pi x/a)] \cdot[\cos(n-1)(\pi y/b) - \cos(n+1)(\pi y/b)] \tag{6}$$

$$Q_y = \sum\sum c_{mn}[\cos(m-1)(\pi x/a) - \cos(m+1)(\pi x/a)] \cdot[\sin(n-1)(\pi y/b) - \sin(n+1)(\pi y/b)] \tag{7}$$

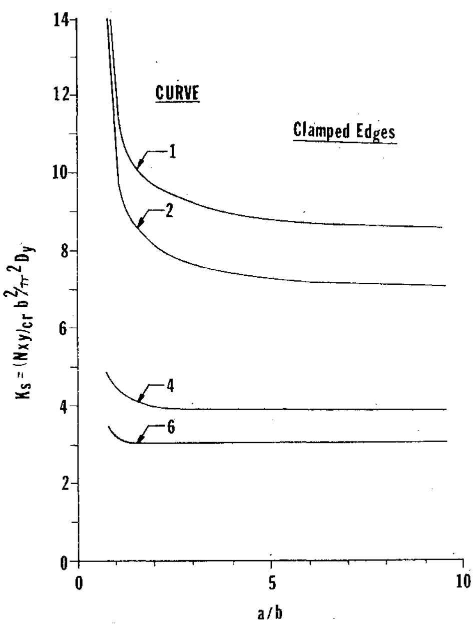

Fig. 1 Buckling curves 1,2,4, and 6 for input data listed in Table 2.

Table 1 **Numerical results for buckling due to edge shear only—clamped edges**

		Panel characteristics			
		C-1	C-2	C-3	C-4
Material properties:					
E_x	10^6 psi	30.0	30.0	30.0	30.0
E_y	10^6 psi	30.0	30.0	3.0	30.0
G_{xy}	10^6 psi	11.5	11.5	2.07	11.5
ν_{xy}		0.30	0.30	0.30	0.30
ν_{yx}		0.30	0.30	0.03	0.30
G_{xz}	10^6 psi	0.038	∞	∞	∞
G_{yz}	10^6 psi	0.038	∞	∞	∞
Geometric parameters:					
a	in.	40.0	10.0	20.0	10.0
b	in.	20.0	10.0	20.0	10.0
t	in.	0.0075	0.10	0.10	0.10
c	in.	0.25	N.A.[a]	N.A.	N.A.
R	in.	∞	31.8	∞	∞
Buckling loads:					
$(N_{xy})_{cr}$	lb/in				
Present work		1762	5459	303	3630
Others		1780	6340	318	3830

[a] N.A. denotes not applicable.

The Galerkin method is applied by substituting these assumed solutions into

$$\int_0^b \int_0^a H_1(w, Q_x, Q_y)(\partial w/\partial a_{mn})\,dxdy = 0 \tag{8}$$

and two analogous equations in which H_1, w, a_{mn} are replaced by H_2, Q_x, b_{mn} and H_3, Q_y, c_{mn}. After performing the integrations, one obtains a doubly infinite set of linear, algebraic equations in terms of m and n. For the simply supported case, the resulting equations are combined into a single equation in the normal-deflection coefficient a_{mn}, Eq. (18) in Ref. 2. For the clamped case, the resulting equations are matrix equations, which are combined into a single matrix equation in Appendix C of Ref. 3.

The problem is now in the familiar form of an eigenvalue problem. Writing the equation for as many wave numbers as required for convergence and separating even and odd modes, one obtains directly values of the shear load N_{xy}, the minimum of which is the buckling load $(N_{xy})_{cr}$.

Pure Edge Shear: Clamped Edges

The following four sample cases were analyzed using a computer routine written for the CDC 6600 computer:

C-1. A flat sandwich panel with isotropic (stainless steel) facings and core.[4]

Table 2 **Material data used in generating the design curves**

Curve	Material	Reference	D_x/D_y	$\nu_{xy} + (D_{xy}/D_y)$
1	Isotropic	—	1.000	1.000
2	Longitudinal glass cloth-epoxy	1	1.044	0.424
3	Longitudinal boron-epoxy	7	9.043	0.950
4	Transverse boron-epoxy	7	0.110	0.099
5	Longitudinal graphite-epoxy	8	25.0	1.248
6	Transverse graphite-epoxy	8	0.040	0.0499

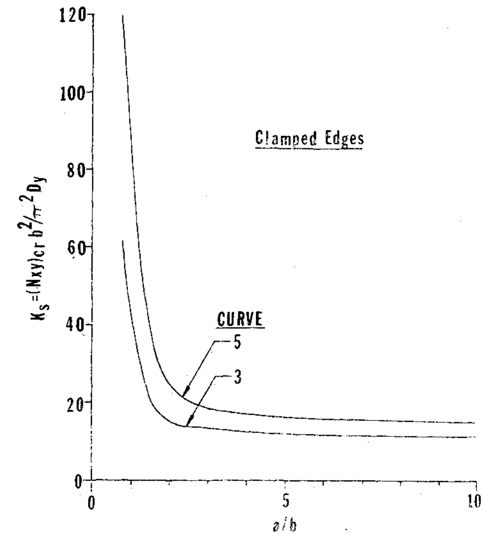

Fig. 2 Buckling curves 3 and 5 for input data listed in Table 2.

C-2. A thin, homogeneous, isotropic panel (aluminum).[5]

C-3. A flat, thin, laminated, orthotropic panel (boron-epoxy).[6]

C-4. A flat, thin, homogeneous, isotropic panel.[5]

The appropriate material properties, geometric data, and results are presented in Table 1.

In Case C-1, the present analysis agrees very well (within 1.0%) with the theoretical analysis in Ref. 4.

For Case C-2, this analysis predicts a somewhat lower buckling load than that reported by Batdorf et al.[5] However, their result is based on an extrapolation of data for infinite-aspect-ratio clamped panels and finite-aspect-ratio panels with simply-supported edges. To determine the range of inaccuracy present in the Ref. 5 analysis, a complete set of data for this case was analyzed, and it was found that the two sets of results agreed very well for panels of large aspect ratios but differed by as much as 13% for aspect ratios ≈ 1. Because of the more exact manner in which the curves and data are generated in the present analysis, it is believed to be a better guide in the design of panels having clamped edges.

For Case C-3, the dimensionless parameters were specialized to those given in Table 1. The five-term Galerkin solution compared reasonably well with the five-term

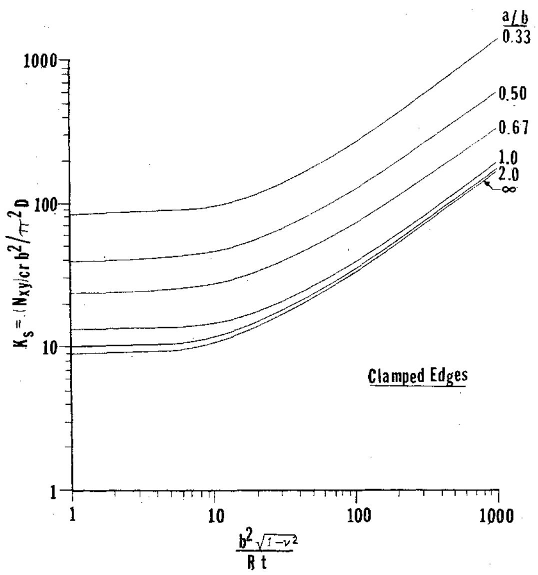

Fig. 3 Design curves for isotropic panels with clamped edges.

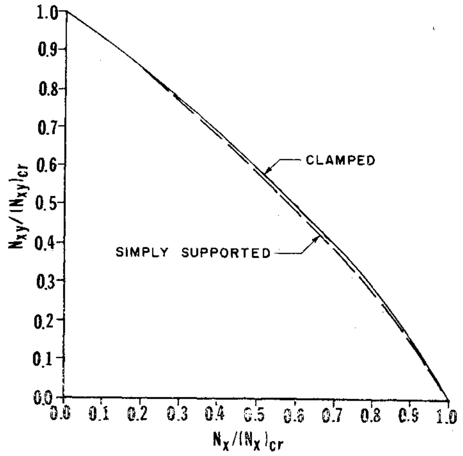

Fig. 4 Interaction curves for combined shear and axial compression for Case I of Ref. 2.

Rayleigh-Ritz solution (within 5%).[6] Some effects from the differences in assumed deflection functions and boundary conditions (see Case C-4) are seen in the slightly lower buckling load predicted by the current analysis. It was also found that by increasing the problem size from a five-term solution to an eight-term solution, the predicted buckling load decreases by 2.5%.

Case C-4 was analyzed as a result of the behavior seen in Case C-2. In this case, a similar behavior was noted, in that a very good agreement was seen in the case of panels having a very large aspect ratio and differing by as much as 10% for an aspect ratio of one. To determine the source of the difference in this case and possibly that in Case C-2, the original analysis[5] for this case was reviewed. That analysis was solved by the Lagrange multiplier method; however, explicit boundary conditions for the displacement components u and v were not presented. It is likely that they were zero at all boundaries, in which case the boundary conditions used here are slightly more flexible. This would account for the slightly lower predicted buckling load and would tend to become less significant as the panel aspect ratio increases, which is in agreement with the observed behavior. Practical shear panels have boundary conditions that fall somewhere between those of this analysis and those of Ref. 5.

In general, the buckling load depends upon twelve material and geometric parameters; thus, it is impractical to present results of a general nature. However, for the case of a flat panel without shear flexibility, the buckling load can be expressed in the dimensionless form, $K_s = (N_{xy})_{cr}b^2/\pi^2 D_y$, as a function of the dimensionless geometric parameter a/b and the two dimensionless material parameters, D_x/D_y and $\nu_{xy} + (D_{xy}/D_y)$. Using the Table 2 parameters for various composite materials, the design curves in Figs. 1 and 2 for clamped edges were calculated. For curved isotropic rectangular panels, the dimensionless design curves of Fig. 3 were generated.

Combined Shear and Axial Compression

For this combined-loading situation, only typical interaction curves were generated. These are for the glass-cloth-reinforced plastic facing, hexagonal-cell aluminum honeycomb-core panels denoted as Case I in Ref. 2. For simply-supported and clamped edges, these curves are shown in Fig. 4.

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Blockage Corrections for Large Bluff Bodies near a Wall in a Closed Jet Wind Tunnel

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Nomenclature

B	= cross section area of wake
C	= cross section area of wind tunnel
D	= drag
C_D	= drag coefficient D/qS
H, p_∞, U	= total head, static pressure and velocity of undisturbed stream
k^2	= base pressure parameter, $1\text{-}C_{p_b}$
K^2	= $(k^2 - 1) = -C_{p_b}$
m	= B/S
p	= static pressure
p_b	= base pressure
C_p	= pressure coefficient $(p\text{-}p_\infty)/q$
q	= dynamic pressure of the undisturbed stream
S	= reference area of the model

Subscripts

c	= suffix denoting effective or corrected values
cc	= suffix denoting the unconstrained values far from the wall

Introduction

SINCE 1965, when a theory for the blockage effects on bluff bodies including stalled wings was published by Maskell,[1] there have been several attempts to determine the range of validity of the theory as well as other limitations if any.[2,3] There seems to be agreement now that the theory is valid up to blockages of about 8% (Ref. 4) in uniform wind beyond which, Melbourne,[2] for example, finds the similarity hypothesis $(p - p_b)/(H - p_b)$, from which Maskell worked out the blockage corrections, no longer valid. There was also clear indication that the con-

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